

The Information Content of Short-Term Options*

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May 6, 2019

Abstract

We exploit a recent development in the S&P 500 index option market to compute the weekly implied variance. We show that the weekly implied variance is a strong predictor of the weekly realized variance. In an encompassing regression test, it crowds out the information content of the monthly implied variance. Further tests reveal that the weekly implied variance outperforms not only the monthly implied variance but also recently proposed time series models of realized variance. This result holds both in- and out-of-sample and the forecast accuracy gains are significant.

JEL classification: G11, G12

Keywords: Implied Variance, Predictability, Realized Variance, Weekly Options

*We are grateful to an anonymous reviewer, Torben Andersen, Fabian Hollstein, Marcel Prokopczuk, Rogier Quaedvlieg, Viktor Todorov and participants at the American Economic Association (2018), the British Accounting and Finance Association (2018), the Western Economic Association International Conference (2018), the Econometrics and Financial Data Science Workshop (2017), the EFiC Conference in Banking and Finance (2017), the International Conference on Computational and Financial Econometrics (2017), the Financial Econometrics Conference at the University of Lancaster (2018), Leibniz University Hannover, Norwich Business School, the University of Glasgow, the University of Kent and the University of Stirling for helpful comments and discussions.

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1 Introduction

Several studies, e.g. [Jiang and Tian \(2005\)](#), [Carr and Wu \(2009\)](#) and [Busch et al. \(2011\)](#), document that the variance implied by monthly options is a strong predictor of the monthly realized variance.¹ While this fact is generally well-accepted, we know relatively little about the forecasting power of short-term implied variance for short-term realized variance. Although it is tempting to speculate that the same findings will hold for the shorter horizon, there are a couple of reasons to suggest otherwise. First, the literature routinely discards short-term options on the grounds that they are illiquid and noisy. Consequently, their information content is assumed to be limited. Second, for the sample period of most existing studies, e.g. [Jiang and Tian \(2005\)](#) and [Busch et al. \(2011\)](#), there were very few short-term options available in the market, making it challenging to obtain a long enough time series of implied variance. This limitation is important because the statistical tests may lack power in a short sample. As a work around this issue, some studies, e.g. [Blair et al. \(2001\)](#), [Pong et al. \(2004\)](#) and [Kourtis et al. \(2016\)](#), have resorted to using the monthly implied variance series to predict short-term realized variance. Alas, this approach introduces a mismatch between the maturity of the options and the forecasting horizon. It is unclear how big of an issue this disconnect may be.

In this paper, we exploit an important innovation in the S&P 500 index option market, namely the launch of weekly option contracts (weeklies) by the Chicago Board of Options Exchange (CBOE) in 2005, to study the predictability of weekly realized variance. Essentially, these option contracts allow market participants to better manage their short-term risk, e.g. the weekly realized variance. [Andersen et al. \(2017\)](#) document that weeklies account for nearly 50 % of the total trading volume in the S&P 500 index options in 2015,

¹See also the survey by [Poon and Granger \(2003\)](#).

indicating that these options are quite liquid. We use the [Bakshi et al. \(2003\)](#) estimator to compute the weekly option implied variance and analyze its predictive power for the weekly realized variance estimated using 5-minute sampled data on the S&P 500 index. In a regression of the daily time series of the weekly realized variance on a constant and the lagged weekly implied variance, we obtain a statistically significant slope estimate and a high predictive power ($Adj\ R^2 = 64.9\%$). In an effort to understand the channel through which this predictability arises, we decompose the realized variance into its continuous and jump components ([Barndorff-Nielsen, 2002](#)). We find that the weekly implied variance predicts both components. Next, we evaluate the incremental information content of the monthly implied variance relative to that of the weekly implied variance. To this end, we formulate and estimate an encompassing model. The regression results suggest that the weekly implied variance crowds out the orthogonalized monthly implied variance series. Furthermore, the forecasting performance of the weekly implied variance is significantly better than that of time series models of the heterogeneous autoregressive (HAR) realized variance family. This set of results holds both in- and out-of-sample.

We run a battery of tests to assess the robustness of our results. The findings are unaffected when using a sampling frequency of 1-minute to estimate the realized variance. Our conclusions are also robust to the method of interpolation used to compute the implied variance series. Furthermore, the key findings are not driven by the choice of the implied variance estimator. Indeed, we obtain qualitatively similar results when using the [Britten-Jones and Neuberger \(2000\)](#) implied variance estimator. Finally, our findings are qualitatively similar if we extend the sample period back to 1996 and adopt a non-overlapping monthly sampling scheme.

Our research relates to the stream of the literature that uses the implied variance

of a given maturity to predict the realized variance of a shorter horizon. [Blair et al. \(2001\)](#), [Pong et al. \(2004\)](#) and [Kourtis et al. \(2016\)](#) are relevant studies along those lines. We confirm their finding that the monthly implied variance predicts the weekly realized variance. However, we find that there are significant gains in forecasting accuracy once the maturity of the option contract and the forecasting horizon are aligned. In fact, our encompassing regression estimates suggest that the weekly implied variance crowds out the monthly implied variance. To the best of our knowledge, we are the first to document this result at the short horizon.

Our study also adds to the broader literature on the predictability of realized variance. [Corsi \(2009\)](#) proposes the HAR model and documents its superior performance relative to the random walk model. [Andersen et al. \(2007\)](#) decompose the historical variance terms of the HAR model into continuous and jump components. [Patton and Sheppard \(2015\)](#) propose an extension that separately uses positive and negative semivariances. [Bollerslev et al. \(2016\)](#) extend the HAR model to account for heteroskedastic measurement errors in realized variance. Empirically, we show that the weekly implied variance provides significantly more accurate forecasts of short-term risk than the HAR model and its aforementioned extensions.

The remainder of this paper proceeds as follows. Section 2 introduces the methodology and the dataset. Section 3 discusses the performance of the weekly implied variance relative to that of the monthly implied variance. Section 4 extends the analysis to various time series models of the HAR family. Section 5 presents the robustness checks. Finally, Section 6 concludes.

2 Methodology and Data

This section introduces the methodology used to construct the main variables used in our analysis. It then presents the dataset.

2.1 Methodology

2.1.a Realized Variance

Our paper focuses on the predictability of next week's realized variance. We start with the definition of the intraday return:

$$r_{j,k} = \log \left(\frac{S_{j,k}}{S_{j,k-1}} \right) \quad (1)$$

where $r_{j,k}$ denotes the intraday return at the end of the k^{th} intraday interval of day j . $S_{j,k}$ and $S_{j,k-1}$ are the asset prices at the end of the k^{th} and $(k-1)^{th}$ intraday interval of trading day j , respectively.

We compute the (annualized) weekly realized variance as follows:

$$RV_{t+7}^w = 52 \times \sum_{j=0}^{N_{t+7}^w-1} \sum_{k=1}^m r_{t+7-j,k}^2 \quad (2)$$

where RV_{t+7}^w is the (annualized) weekly realized variance for the week ending on day $t+7$. The number 52 indicates that the realized variance estimate is annualized. N_{t+7}^w denotes the number of trading days during the week ending on day $t+7$. There are m returns observed on each trading day. The case where $k=1$ corresponds to the overnight return.²

²It is standard in the literature to account for the overnight returns. See [Bollerslev et al. \(2009\)](#), [Drechsler and Yaron \(2011\)](#) and [Bekaert and Hoerova \(2014\)](#), among others.

2.1.b Implied Variance

The literature often uses the [Britten-Jones and Neuberger \(2000\)](#) estimator to proxy for the risk-neutral expectation of the total variation of returns. However, this estimator captures the risk-neutral expectation of the continuous variation which is equal to the total variation of returns only if the underlying return process does not jump. This result arises because the total variation is the sum of the continuous and jump variations. In the presence of jumps, the [Britten-Jones and Neuberger \(2000\)](#) estimator is a biased estimator of the risk-neutral expectation of the future total variation and the magnitude of this bias increases with the contribution of jumps to the total variation of returns ([Du and Kapadia, 2013](#)).

[Andersen et al. \(2007\)](#) and [Lee and Mykland \(2008\)](#), among others, use non-parametric statistical tests to show that the S&P 500 index jumps.³ [Du and Kapadia \(2013\)](#) conduct an extensive simulation exercise to assess the impact of jumps on the implied variance and recommend the [Bakshi et al. \(2003\)](#) estimator as a jump-robust estimator of implied variance. Heeding on their recommendation, we use the [Bakshi et al. \(2003\)](#) formula to compute the implied variance:⁴

$$IV_t^\tau = \frac{360}{\tau} \left[e^{rf_t \frac{\tau}{360}} QUAD_t - \left(e^{rf_t \frac{\tau}{360}} - 1 - \frac{e^{rf_t \frac{\tau}{360}}}{2} QUAD_t \right)^2 \right] \quad (3)$$

where

$$QUAD_t = \int_0^{S_t} \frac{2 \left(1 + \ln \left[\frac{S_t}{K} \right] \right)}{K^2} P_t(\tau, K) dK + \int_{S_t}^{+\infty} \frac{2 \left(1 - \ln \left[\frac{K}{S_t} \right] \right)}{K^2} C_t(\tau, K) dK \quad (4)$$

³The documented jumps are not spuriously induced by the fact that the index is not directly tradable. [Prokopczuk and Wese Simen \(2016\)](#) show that the liquid S&P 500 E-Mini futures contract, which is tradable, also jumps.

⁴To make our analysis more comparable to prior studies, we also consider the [Britten-Jones and Neuberger \(2000\)](#) implied variance. Section 5.3 presents these results. Note, however, that these findings should be interpreted cautiously, keeping in mind that this specific estimator is not robust to jumps in the underlying return process.

IV_t^τ is the (annualized) implied variance of time to maturity τ (expressed in days) observed on day t . Throughout this paper, we use the expressions “weekly” and “monthly” to denote the case where $\tau = 7$ and $\tau = 30$ calendar days, respectively. rf_t is the τ -day (annualized) discount rate on day t . S_t is the underlying price on day t . $P_t(\tau, K)$ and $C_t(\tau, K)$ denote the price on day t of the European put and call options of time to maturity τ and strike price K , respectively. Note that the formula in Equation (4) involves only out-of-the-money (OTM) options. For each option maturity available on that day, we compute the [Black and Scholes \(1973\)](#) implied volatility for all OTM options. We then average the OTM implied volatility estimates of the same maturity. Equipped with this average implied volatility, denoted σ , we define the variables $K_{t,L}$ and $K_{t,U}$ as follows:

$$K_{t,L} = S_t e^{-8\sigma_t} \quad (5)$$

$$K_{t,U} = S_t e^{8\sigma_t} \quad (6)$$

where σ_t is the average implied volatility at time t of all OTM options of the same maturity.

Similar to [Carr and Wu \(2009\)](#), we linearly interpolate the implied volatilities for 2,000 equally spaced strike prices between $K_{t,L}$ and $K_{t,U}$ defined in Equations (5) and (6), respectively. In practice, the strike prices traded in the market do not completely span the interval starting at $K_{t,L}$ and ending at $K_{t,U}$, raising the question of extrapolation. We follow [Jiang and Tian \(2005\)](#) and [Carr and Wu \(2009\)](#), among others, and perform the nearest neighbourhood extrapolation. To be precise, for strike prices greater (lower) than $K_{t,L}$ ($K_{t,U}$) but lower (higher) than the lowest (highest) strike available in the market, we use the implied volatility associated with the lowest (highest) strike available in the

market. Next, we map the grid of 2,000 implied volatilities into [Black and Scholes \(1973\)](#) OTM option prices. Finally, we use the trapezoidal rule to numerically evaluate the integrals in Equation (4) and compute the implied variance as in Equation (3).⁵

We repeat the steps above for all maturities observed on that day to obtain the term-structure of implied variances. From this term-structure, we linearly interpolate the implied variance of weekly (IV^w) and monthly (IV^m) horizons. We emphasize that we only interpolate between maturities and never extrapolate since this could introduce spurious spikes in the constant maturity implied variance series.⁶

2.2 Data

We obtain high-frequency data on the S&P 500 index from Thomson Reuters Tick History (TRTH) to build the realized variance series. Our interest in high-frequency data, as opposed to daily data, is motivated by the studies of [Andersen and Bollerslev \(1998\)](#), [Barndorff-Nielsen and Shephard \(2002\)](#) and [Andersen et al. \(2003\)](#), who recommend the use of intraday data to efficiently measure realized variance. The dataset spans the period from January 1996 to August 2015.⁷ It contains bid and ask quotes pertaining to regular business hours, i.e. from 8:30 AM to 3:00 PM (Chicago Time). Similar to [Bollerslev et al. \(2009\)](#) and [Bollerslev and Todorov \(2011\)](#), among others, we use a 5-minute sampling frequency.⁸ At the end of each 5-minute interval, we use the most recent mid-quote price

⁵Note that by using options with strike prices ranging from $K_{t,L}$ to $K_{t,U}$, we essentially truncate the integrals in Equation (4). This choice is consistent with earlier work, e.g. [Carr and Wu \(2009\)](#). In a simulation setting, [Jiang and Tian \(2005\)](#) show that the truncation error is negligible if the truncation points are more than two standard deviations from the current underlying price.

⁶As a robustness check, we do not linearly interpolate the constant maturity contracts but instead simply use the option contracts with maturity closest to the target maturity. On average, the actual maturities of the options are 6 and 27 calendar days for the weekly and monthly horizons, respectively. Our main findings are unchanged, leading us to conclude that the method of interpolation plays a minimal role in our results. See Section 5.2 for further details.

⁷The sample period is dictated by the dataset available from TRTH at the time we started this project.

⁸As a robustness check, we consider a higher sampling frequency of 1-minute and obtain qualitatively similar findings. See Section 5.1 for further details.

to proxy for the closing price of that interval.

To estimate our implied variance series, we obtain end-of-day S&P 500 index options data for the same period from IvyDB OptionMetrics. These options trade on the CBOE and are of the European type. For each trading day and option contract, the database contains information about the bid and ask prices, the open interest, the strike price and the expiration date. The dataset includes weekly and standard option contracts, among others. Generally, weekly options expire on the Friday of each week, except the third Friday of each month when the standard options expire.⁹ Figure 3 of Andersen et al. (2017) reveals a rapid growth in the trading volume of the weekly contracts from less than 5 % of the total S&P 500 index option volume during the first few years of trading to 50 % towards the end of our sample.¹⁰

Although the OptionMetrics dataset spans the same time period as the TRTH dataset, we face two sample limitations that require us to start our main analysis in March 2008. The first limitation is forced upon us by the fact that weekly options on the S&P 500 index were launched in October 2005. As a result, we can only analyze the period beginning from that point onwards. The second limitation is driven by the way OptionMetrics reports the closing option and underlying prices. Prior to the 5th of March 2008, the OptionMetrics dataset records derivatives prices at 3:15 PM (the market closing) whereas the underlying spot price is recorded at 3:00 PM, introducing a bias in studies that rely

⁹At the time we started the project, the term-structure of weekly options included up to 12 maturities. For further information about weekly options, we refer the interested reader to the following webpage: [http : //www.cboe.com/micro/weeklys/introduction.aspx](http://www.cboe.com/micro/weeklys/introduction.aspx). For an up-to-date list of weekly option contracts on offer, we direct the reader to the following link: [http : //www.cboe.com/micro/weeklys/availableweeklys.aspx](http://www.cboe.com/micro/weeklys/availableweeklys.aspx).

¹⁰Absent official data on the identity/profile of market participants who trade weekly options, it is difficult to definitely ascertain their trading motives. Andersen et al. (2017) do not find a significant change in the trading activity of these contracts around important macroeconomic news announcements. This finding suggests that speculation does not seem to be the main driver of their trading activity. This leaves open the possibility that the increased trading activity in weeklies is primarily driven by a desire to improve short-term risk management.

on synchronous observations of the derivatives and underlying prices. To remediate to this issue, we focus on the sample period extending from the 5th of March 2008 to the end of August 2015.^{11,12}

We process the option data as follows. We discard observations with missing or zero prices. We implement this filter on bid and ask prices separately. In doing so, we aim to tackle the concern that our dataset includes contracts which are not actively quoted. As is standard in the literature ([Carr and Wu, 2009](#)), we compute the mid-quote price of the option, which we refer to as the option price. Next, we remove all option observations that are in-the-money. We take this step because the computation of the implied variance only involves OTM option prices (see Equation (4)). Furthermore, we download the discount rates from OptionMetrics. These discount rates are based upon the London Interbank Offered Rates (LIBOR) and the Eurodollar futures. For each trading day and option contract, we linearly interpolate the discount rate of the same time to maturity as the option contract. We then match the discount rates with the panel of options data. We also match the time series of the daily S&P 500 index prices and that of the dividend yield, both obtained from OptionMetrics, with the panel of options data.

Our analysis involves daily observations of all key variables. Table 1 presents the summary statistics of the main series. The weekly and monthly implied volatility have average (annualized) values of 21.345% and 21.837%, respectively.

¹¹One may argue that we should start our sample period at a later date, e.g. in 2011 as in [Andersen et al. \(2017\)](#), to allow the trading activity in weekly options to pick up. We also considered this alternative starting date and reached qualitatively similar results. If one holds the view that our current sample period includes illiquid weeklies, then this low trading activity should work against the predictive power of the weekly options. Viewed in this way, the gains in forecasting performance documented in our paper represent a “worst case” scenario.

¹²Once a month, we observe a standard option contract with 7 days left to expiration. Thus, if one changes the sampling frequency to the monthly level, it is possible to extend the sample period back to 1996 and repeat the main analyses. Section 5.4 explores this possibility and shows that the main results are robust to this change. We thank the reviewer for suggesting this analysis.

3 Weekly vs. Monthly Implied Variance

This section starts by examining the in-sample predictive power of the weekly implied variance for next week’s realized variance and compares it to that of the monthly implied variance. It then explores the channels through which this predictability arises.

3.1 The Information Content of the Weekly Implied Variance

3.1.a Univariate Evidence

We begin by evaluating the information content of implied variance for the week ahead realized variance. To this end, we estimate the following [Mincer and Zarnowitz \(1969\)](#) regression:

$$RV_{t+7}^w = \alpha + \beta IV_t^x + \epsilon_{t+7} \quad (7)$$

where α is the intercept. β denotes the slope parameter. IV_t^x is the implied variance on day t of time to maturity x , where x can be the weekly (w) or monthly (m) maturity. ϵ_{t+7} is the residual of the regression at $t + 7$. If the implied variance is informative about the future weekly realized variance, then the slope parameter will be significantly different from 0.

Starting with the monthly implied variance, Table 2 reports a positive and statistically significant ($t - statistic = 6.676$) slope estimate of 0.785. The explanatory power associated with this regression, $Adj R^2 = 61.6\%$, confirms that the monthly implied variance predicts the weekly realized variance.

Turning to the weekly implied variance, we can see that it predicts the future realized variance with a slope estimate of 0.713 ($t - statistic = 7.240$). The corresponding

predictive power is equal to 64.9%. Several points are worth highlighting. First, the explanatory power of this regression model is higher than that of the model which relies on the monthly implied variance. This result indicates that the weekly implied variance has superior predictive ability for short-term risk than the often-used monthly implied variance. Second, the slope estimates obtained from these univariate regressions are similar and significantly different from 1. While the earlier literature interpreted a slope that is significantly different from 1 as evidence against the expectations hypothesis, [Chernov \(2007\)](#) and [Prokopczuk and Wese Simen \(2014\)](#) point out that this result can arise in a setting where the variance risk premium is time-varying.¹³ It is thus useful to analyze the average variance risk premium, defined as the difference between the implied variance and the contemporaneously estimated realized variance ([Bollerslev et al., 2009](#)), of each maturity. Table 1 reports an average of 2.280% with a volatility of 4.659% for the (annualized) weekly variance risk premium and an average of 2.300% with a volatility of 3.107% for the (annualized) monthly variance risk premium. Thus, there is evidence of a non-zero and time-varying variance risk premium in both the weekly and monthly implied variances.

3.1.b Multivariate Evidence

The preceding analysis shows that, when used alone, the weekly and monthly implied variance predict the future weekly realized variance. However, it does not directly shed light on the incremental information content of these two predictors.

¹³Note that the high persistence of the implied variance series along with the short sample could also easily generate a spurious mean-reversion leading to a slope coefficient lower than 1. We thank the reviewer for providing this insight.

To address this question, we estimate the following model:

$$RV_{t+7}^w = \alpha + \beta IV_t^w + \gamma \hat{\nu}_t^{m/w} + \epsilon_{t+7} \quad (8)$$

where α is the intercept. β and γ are the slope parameters. IV_t^w is the weekly implied variance on day t . $\hat{\nu}_t^{m/w}$ is the estimated residual at time t of the regression of the monthly implied variance on a constant and the weekly implied variance (see Equation (9)):¹⁴

$$IV_t^m = \phi_0 + \phi_1 IV_t^w + \nu_t^{m/w} \quad (9)$$

The last row of Table 2 reports that the slope estimate associated with the weekly component is significant ($t - stat = 7.025$), whereas that of the orthogonalized monthly implied variance is not ($t - stat = 0.631$). Moreover, the explanatory power of the encompassing model ($Adj R^2 = 65.0\%$) is very similar to that of the univariate model which uses the weekly implied variance as a forecasting variable ($Adj R^2 = 64.9\%$). Taken as a whole, the results suggest that the orthogonalized monthly implied variance does not add to the predictive power of the weekly implied variance. This set of findings is important because empirical studies routinely discard short-term options data on the grounds that they are noisy and thus uninformative. Our results caution that, by following this approach, one throws away valuable information about short-term risk. These findings are also interesting given the growing practice of using the monthly implied variance to predict the short-term realized variance. Our evidence reveals that this methodology may

¹⁴In an earlier version of the paper, we directly included the monthly implied variance rather than its orthogonalized component with respect to the weekly implied variance. While this analysis led to qualitatively similar conclusions, it was vulnerable to the concern that the results are difficult to interpret given the high correlation between the two implied variance series. By using the orthogonalized component of the monthly implied variance, we assuage this concern. We thank the reviewer for suggesting this analysis.

not be the best way of modeling the short-term realized variance.

3.2 Dissecting the Predictability

Having established the in-sample predictive power of the weekly implied variance for weekly realized variance, we now seek to further explore the channel through which this predictability arises.

3.2.a Framework

Our starting point is the theory of quadratic variation ([Barndorff-Nielsen and Shephard, 2002](#)) which posits that the realized variance of asset returns can be decomposed into components linked to (i) the significant continuous variation and (ii) the significant jump variation of the asset returns. More formally, we have:

$$RV_{t+7}^w = C_{t+7}^w + J_{t+7}^w \quad (10)$$

where C_{t+7}^w and J_{t+7}^w are the significant weekly continuous and jump variations of the asset returns computed over the week ending on day $t + 7$, respectively.

This insight suggests that there are two channels through which short-term implied variance may be informative about next week's realized variance. The first possibility is that the weekly implied variance contains information about the significant continuous variation of returns. The second possibility is that the weekly implied variance is informative about the significant jump variation.

[Barndorff-Nielsen and Shephard \(2002\)](#) propose the bipower variation as an estimator of the continuous variation of asset returns. [Andersen et al. \(2012\)](#) subsequently establish that the *MedRV* estimator has better properties than the bipower variation. Thus, we

use the *MedRV* estimator of the continuous variation of returns:

$$CV_{t+7}^w = \underbrace{\frac{52 \times m\pi}{(6 - 4\sqrt{3} + \pi)(m - 2)} \sum_{j=0}^{N_{t+7}^w - 1} \sum_{k=3}^m \text{median}(|r_{t+7-j,k}|, |r_{t+7-j,k-1}|, |r_{t+7-j,k-2}|)}_{MedRV \text{ Estimator}}^2 \quad (11)$$

where $\text{median}(\cdot)$ is the median operator and all other variables are as previously defined. Next, we modify the test statistic presented in [Huang and Tauchen \(2005\)](#) that relies on the bipower variation and the realized quarticity to take advantage of the more robust estimators of the continuous variation (*MedRV*) and realized quarticity (*MedRQ*). We are thus able to compute the significant continuous and jump variations:

$$z_{t+7}^w = m^{1/2} \left(\frac{\frac{RV_{t+7}^w - CV_{t+7}^w}{RV_{t+7}^w}}{\sqrt{0.96 \max(1, \frac{MedRQ_{t+7}^w}{(CV_{t+7}^w)^2})}} \right) \quad (12)$$

$$MedRQ_{t+7}^w = A_w \times \sum_{j=0}^{N_{t+7}^w - 1} \sum_{k=3}^m \text{median}(|r_{t+7-j,k}|, |r_{t+7-j,k-1}|, |r_{t+7-j,k-2}|)^4 \quad (13)$$

$$A_w = \frac{52^2 \times 3m^2\pi}{(72 - 52\sqrt{3} + 9\pi)(m - 2)} \quad (14)$$

$$C_{t+7}^w = I_{z_{t+7}^w > \phi_{1-\alpha}} CV_{t+7}^w + I_{z_{t+7}^w \leq \phi_{1-\alpha}} RV_{t+7}^w \quad (15)$$

$$J_{t+7}^w = I_{z_{t+7}^w > \phi_{1-\alpha}} (RV_{t+7}^w - CV_{t+7}^w) \quad (16)$$

where $\phi_{1-\alpha}$ is the critical value from the cumulative standard normal distribution at confidence level $1 - \alpha$. I is the indicator function. Similar to [Andersen et al. \(2007\)](#), we employ $\alpha = 99.9\%$ throughout this paper.

The middle panel of Figures 1 to 3 show the dynamics of the realized variance, the significant continuous variation and the significant jump variation of weekly horizon, respectively.

3.2.b Significant Continuous Variation

We regress the time series of the significant continuous variation on a constant and the lagged implied variance series:

$$C_{t+7}^w = \alpha + \beta IV_t^x + \epsilon_{t+7} \quad (17)$$

where all variables are as previously defined.

Panel A of Table 3 shows that, in univariate regressions, each maturity of the implied variance predicts the significant continuous component of the realized variance. This conclusion is borne out by the significant slope estimates in the univariate regressions. Similar to our analysis of the realized variance, we find that the weekly implied variance boasts the higher predictive power ($Adj R^2 = 63.6\%$) of the two variables. The encompassing model yields an $Adj R^2$ of 63.7% which is very close to that of the univariate model based on the weekly implied variance alone.

3.2.c Significant Jump Variation

We now estimate the following forecasting regression:

$$J_{t+7}^w = \alpha + \beta IV_t^x + \epsilon_{t+7} \quad (18)$$

where all variables are as previously defined.

Panel B of Table 3 documents that it is harder to accurately model the significant jump variation than the significant continuous variation. This conclusion is evidenced by the lower explanatory power for the significant jump variation compared to that of

the significant continuous variation (see Panel A).¹⁵ We can see that each maturity of the implied variance individually predicts the significant weekly jump variation with very similar explanatory power. However, the encompassing regression reveals that the weekly implied variance crowds out the orthogonalized monthly implied variance.

4 Implied Variance vs. Time Series Models

The previous section shows that the weekly implied variance is superior to the monthly implied variance in-sample. However, it is not clear how it compares to other sophisticated time series models that have been recently proposed in the literature. This section starts by presenting the competing models. Next, it assesses their in- and out-of-sample predictive performance.

4.1 Introducing the Competing Models

HAR We use the heterogeneous autoregressive (HAR) realized variance model ([Corsi, 2009](#)) as our benchmark:

$$RV_{t+7}^w = \alpha + \beta RV_t^d + \gamma RV_t^w + \delta RV_t^m + \epsilon_{t+7} \quad (19)$$

where α is the intercept. β , γ and δ are the slope parameters. RV_t^d and RV_t^m are the (annualized) daily and monthly realized variance at time t , respectively. These series are computed using the estimators below:

$$RV_t^d = 252 \times \sum_{k=1}^m r_{t,k}^2 \quad (20)$$

¹⁵Using various time series models, [Busch et al. \(2011\)](#) document a similar result at the monthly horizon.

$$RV_t^m = 12 \times \sum_{j=0}^{N_t^m-1} \sum_{k=1}^m r_{t-j,k}^2 \quad (21)$$

where the numbers 252 and 12 serve to annualize the daily and monthly realized variance estimates, respectively. N_t^m is the number of trading days in the calendar month ending on day t .

CHAR Building on the work of [Andersen et al. \(2007\)](#), we also consider the continuous heterogeneous autoregressive (CHAR) model that replaces each historical variance in the HAR model with the continuous variation of the corresponding horizon:

$$RV_{t+7}^w = \alpha + \beta CV_t^d + \gamma CV_t^w + \delta CV_t^m + \epsilon_{t+7} \quad (22)$$

where CV_t^d and CV_t^m are the (annualized) daily and monthly continuous variations at time t , respectively:

$$CV_t^d = \frac{252 \times m\pi}{(6 - 4\sqrt{3} + \pi)(m - 2)} \sum_{k=3}^m \text{median}(|r_{t,k}|, |r_{t,k-1}|, |r_{t,k-2}|)^2 \quad (23)$$

$$CV_t^m = \frac{12 \times m\pi}{(6 - 4\sqrt{3} + \pi)(m - 2)} \sum_{j=0}^{N_t^m-1} \sum_{k=3}^m \text{median}(|r_{t-j,k}|, |r_{t-j,k-1}|, |r_{t-j,k-2}|)^2 \quad (24)$$

HAR-J We also analyze the performance of the HAR-J model ([Andersen et al., 2007](#)), which augments the HAR model with the lagged significant daily jump variation:

$$RV_{t+7}^w = \alpha + \beta RV_t^d + \gamma RV_t^w + \delta RV_t^m + \eta J_t^d + \epsilon_{t+7} \quad (25)$$

where α , β , γ , δ and η are parameters to estimate. J_t^d is the (annualized) statistically significant daily jump variation at time t computed as follows:

$$J_t^d = I_{z_t^d > \phi_{1-\alpha}}(RV_t^d - CV_t^d) \quad (26)$$

$$z_t^d = m^{1/2} \left(\frac{\frac{RV_t^d - CV_t^d}{RV_t^d}}{\sqrt{0.96 \max(1, \frac{MedRQ_t^d}{(CV_t^d)^2})}} \right) \quad (27)$$

$$MedRQ_t^d = A_d \times \sum_{k=3}^m \text{median}(|r_{t,k}|, |r_{t,k-1}|, |r_{t,k-2}|)^4 \quad (28)$$

$$A_d = \frac{252^2 \times 3m^2\pi}{(72 - 52\sqrt{3} + 9\pi)(m - 2)} \quad (29)$$

HAR-C-J We also evaluate the forecasting performance of the HAR-C-J model ([Andersen et al., 2007](#)). Essentially, this model decomposes each historical variance in the HAR model into the corresponding significant continuous and jump variations:

$$RV_{t+7}^w = \alpha + \beta C_t^d + \gamma J_t^d + \delta C_t^w + \eta J_t^w + \theta C_t^m + \kappa J_t^m + \epsilon_{t+7} \quad (30)$$

where α , β , γ , δ , η , θ and κ are parameters to estimate. C_t^d and C_t^m are the corresponding daily and monthly significant continuous variation at time t , respectively. J_t^m denotes the (annualized) monthly significant jump variation at time t . We obtain C_t^d , C_t^m and J_t^m as follows:

$$C_t^d = I_{z_t^d > \phi_{1-\alpha}} CV_t^d + I_{z_t^d \leq \phi_{1-\alpha}} RV_t^d \quad (31)$$

$$C_t^m = I_{z_t^m > \phi_{1-\alpha}} CV_t^m + I_{z_t^m \leq \phi_{1-\alpha}} RV_t^m \quad (32)$$

$$J_t^m = I_{z_t^m > \phi_{1-\alpha}}(RV_t^m - CV_t^m) \quad (33)$$

$$z_t^m = m^{1/2} \left(\frac{\frac{RV_t^m - CV_t^m}{RV_t^m}}{\sqrt{0.96 \max(1, \frac{MedRQ_t^m}{(CV_t^m)^2})}} \right) \quad (34)$$

$$MedRQ_t^m = A_m \times \sum_{j=0}^{N_t^m-1} \sum_{k=3}^m \text{median}(|r_{t-j,k}|, |r_{t-j,k-1}|, |r_{t-j,k-2}|)^4 \quad (35)$$

$$A_m = \frac{12^2 \times 3m^2\pi}{(72 - 52\sqrt{3} + 9\pi)(m-2)} \quad (36)$$

SHAR Patton and Sheppard (2015) document the good empirical performance of the semi-variance heterogeneous autoregressive (SHAR) model. Essentially, this model modifies the HAR specification by decomposing each historical variance term into positive and negative semivariance components:¹⁶

$$RV_{t+7}^w = \alpha + \beta SV_t^{d+} + \gamma SV_t^{d-} + \delta SV_t^{w+} + \eta SV_t^{w-} + \theta SV_t^{m+} + \kappa SV_t^{m-} + \epsilon_{t+7} \quad (37)$$

where α , β , γ , δ , η , θ and κ are parameters to estimate. SV_t^{d+} , SV_t^{w+} and SV_t^{m+} are the positive (annualized) daily, weekly and monthly semivariances at time t , respectively. SV_t^{d-} , SV_t^{w-} and SV_t^{m-} are the negative (annualized) daily, weekly and monthly semivariances at time t , respectively. We compute these variables below:

$$SV_t^{d+} = 252 \times \sum_{k=1}^m r_{t,k}^2 I_{r_{t,k} > 0} \quad (38)$$

$$SV_t^{d-} = 252 \times \sum_{k=1}^m r_{t,k}^2 I_{r_{t,k} < 0} \quad (39)$$

$$SV_t^{w+} = 52 \times \sum_{j=0}^{N_t^w-1} \sum_{k=1}^m r_{t-j,k}^2 I_{r_{t-j,k} > 0} \quad (40)$$

$$SV_t^{w-} = 52 \times \sum_{j=0}^{N_t^w-1} \sum_{k=1}^m r_{t-j,k}^2 I_{r_{t-j,k} < 0} \quad (41)$$

$$SV_t^{m+} = 12 \times \sum_{j=0}^{N_t^m-1} \sum_{k=1}^m r_{t-j,k}^2 I_{r_{t-j,k} > 0} \quad (42)$$

¹⁶We have also considered more parsimonious specifications of the SHAR model where we only decompose the historical variance of a specific horizon into positive and negative semivariance components. We found very little to distinguish between these alternative specifications.

$$SV_t^{m-} = 12 \times \sum_{j=0}^{N_t^m-1} \sum_{k=1}^m r_{t-j,k}^2 I_{r_{t-j,k} < 0} \quad (43)$$

4.2 In-Sample Evidence

Table 4 sheds light on the in-sample forecasting performance of each model. Starting with the HAR model, we can see that it yields an explanatory power of 59.8%. The CHAR model yields a comparable explanatory power of 59.6%. The fit of the HAR-J model to the data ($Adj\ R^2 = 59.8\%$) is similar to that of the HAR model. This result arises because the exposure to the significant daily jump variation is not statistically significant. Turning to the HAR-C-J and SHAR models, we can see that they slightly improve on the benchmark HAR model as evidenced by their $Adj\ R^2$ of 60.8% and 60.7%, respectively. This conclusion is consistent with the in-sample finding of [Patton and Sheppard \(2015\)](#).

Comparing the $Adj\ R^2$ of Tables 2 and 4, we can see that the forecasting model which uses the weekly implied variance as sole predictor achieves the highest explanatory power ($Adj\ R^2 = 64.9\%$). This result leads us to conclude that the weekly implied variance performs better than the HAR model and its extensions in-sample.

4.3 Out-of-Sample Evidence

We next investigate whether the in-sample predictability results also extend out-of-sample. To this end, we define the models in Equation (7) based on the IV^w and IV^m as the IVW and IVM models, respectively. We use a rolling window of 4 years of daily data to estimate the forecasting models in Equations (7), (19), (22), (25), (30) and (37).¹⁷

¹⁷A window of 4 years is consistent with the research of [Bollerslev et al. \(2016\)](#) and [Patton and Sheppard \(2015\)](#).

We then use all relevant information available in real-time to generate the conditional expectation of next week’s realized variance. Similar to [Bollerslev et al. \(2016\)](#), we subject these forecasts to the “insanity filter” to guard ourselves against implausible variance forecasts. If the forecast is higher (lower) than the highest (lowest) weekly realized variance observed in the estimation window, we set the forecast to the average weekly realized variance in the estimation window. This filter also enables us to avoid the situation where the variance forecast could be negative.¹⁸

Repeating the steps above for each rolling window, we obtain the time series of the out-of-sample variance forecasts which we then compare to the realized variance. We compute the mean squared error (*MSE*) and quasi-likelihood (*QLIKE*) loss functions below:

$$MSE = \frac{1}{T} \sum_{t=1}^T (RV_{t+7}^w - E_t(RV_{t+7}^w))^2 \quad (44)$$

$$QLIKE = \frac{1}{T} \sum_{t=1}^T \left(\frac{RV_{t+7}^w}{E_t(RV_{t+7}^w)} - \log \frac{RV_{t+7}^w}{E_t(RV_{t+7}^w)} - 1 \right) \quad (45)$$

where T is the total number of out-of-sample forecasts. $E_t(RV_{t+7}^w)$ is the forecast at time t of the variance to be realized at $t + 7$. All other variables are as previously defined.

[Patton \(2011\)](#) shows that these two loss functions are robust to the noise in the realized variance proxy, making them well-suited for our analysis.¹⁹ Table 5 reports the ratio of the loss function [name in row] associated with the model [name in column] over that of the benchmark HAR model. An entry equal to 1 indicates that the model [name in column] does as well as the benchmark HAR model. Entries lower than 1 suggest that the model [name in column] achieves lower average forecasting error than the HAR model.

¹⁸As a robustness check, we remove the filter and obtain similar results.

¹⁹Section 5.1 discusses further the implications of measurement errors.

Conversely, entries that are greater than 1 indicate that the average forecast errors of the model [name in column] are higher than those of the HAR model.

Focusing on the entries reported for the IVW model, we can see that the *MSE* and *QLIKE* ratios are equal to 0.797 and 0.870, respectively. This set of numbers reveals that a forecasting model based on the weekly implied variance reduces the forecasting errors of the benchmark HAR by 20.3 % (*MSE*) and 13.0 % (*QLIKE*). Looking across the competing models, we observe that their performance is generally inferior to that of the weekly implied variance.

We formally test the null hypothesis that the average forecast loss associated with the weekly implied variance is equal to or greater than that of the best forecasting model among its competitors (IVM, HAR, CHAR, HAR-J, HAR-C-J and SHAR). The alternate hypothesis is that the weekly implied variance delivers lower average forecast errors than its competitors. To implement this test, we modify the Reality Check test of [White \(2000\)](#) as in [Bollerslev et al. \(2016\)](#). In our empirical implementation, we use the stationary bootstrap of [Politis and Romano \(1994\)](#) with 9,999 re-samplings and an average block length of 10.²⁰ The null hypothesis is rejected at the 5% significance level with *p*-values of 1.9% and 1.6% for the *MSE* and *QLIKE* loss functions, respectively. We thus conclude that the weekly implied variance achieves significantly lower forecasting errors relative to its competitors.

5 What About ...

This section presents several robustness checks. First, we explore whether our results are affected by measurement errors in the historical variance estimates. Second, we

²⁰We experimented with different block lengths and obtained very similar results.

examine the impact of potential errors induced by the method of interpolation used to obtain the implied variance series. Third, we assess the sensitivity of our findings to the estimator of implied variance. Fourth, we consider a longer sample period that dates back to 1996. Fifth, we analyze the predictability of quarterly realized variance.

5.1 Noise in Realized Variance?

[Bollerslev et al. \(2016\)](#) recently propose a forecasting model that extends the HAR model by taking into account the measurement errors in the historical variance. These errors arise from the fact that the historical variance is not directly observable. Thus, one needs to estimate the historical variance before using it for forecasting, leading to the errors-in-variables problem. The authors introduce a modeling framework, termed HAR–RQ, which aims to capture the heteroskedasticity of the measurement errors and improve the realized variance forecasts:²¹

$$\begin{aligned} RV_{t+7}^w = & \alpha + (\beta + \gamma\sqrt{MedRQ_t^d})RV_t^d + (\delta + \eta\sqrt{MedRQ_t^w})RV_t^w \\ & + (\theta + \kappa\sqrt{MedRQ_t^m})RV_t^m + \epsilon_{t+7} \end{aligned} \quad (46)$$

where α is the intercept. β , γ , δ , η , θ and κ are slope parameters.²²

Using the same methodology as before, we analyze the out-of-sample predictive performance of the HAR–RQ model. Consistent with [Bollerslev et al. \(2016\)](#), Table 5 shows that this model yields more accurate forecasts than the CHAR, HAR–J and HAR–C–

²¹[Bollerslev et al. \(2016\)](#) use a square root specification for the measurement error correction on the basis that it has an imbued robustness. We also experimented with the logarithmic specification and found it to deliver inferior forecasting performance compared to the square root specification. This finding is consistent with the authors’ argument and their own empirical results.

²²[Bollerslev et al. \(2016\)](#) use the realized quarticity as defined in [Barndorff-Nielsen \(2002\)](#) rather than the more robust MedRQ estimator of [Andersen et al. \(2012\)](#). We have also repeated the analysis using the same estimator as the authors. The results were qualitatively similar.

J models. However, its performance is inferior to that of the weekly implied variance, indicating that the short-term implied variance provides more accurate forecasts than the HAR–RQ model. Our untabulated analysis also reveals that the difference in the performance of the two models is statistically significant.

In addition, we consider more parsimonious specifications of the HAR–RQ model in Equation (46), namely: the HAR–RQ–D (Equation(47)), HAR–RQ–W (Equation(48)) and HAR–RQ–M (Equation(49)).

$$RV_{t+7}^w = \alpha + (\beta + \gamma\sqrt{MedRQ_t^d})RV_t^d + \delta RV_t^w + \theta RV_t^m + \epsilon_{t+7} \quad (47)$$

$$RV_{t+7}^w = \alpha + \beta RV_t^d + (\delta + \eta\sqrt{MedRQ_t^w})RV_t^w + \theta RV_t^m + \epsilon_{t+7} \quad (48)$$

$$RV_{t+7}^w = \alpha + \beta RV_t^d + \delta RV_t^w + (\theta + \kappa\sqrt{MedRQ_t^m})RV_t^m + \epsilon_{t+7} \quad (49)$$

Our untabulated analysis reveals that these parameterizations do not outperform the weekly implied variance. For instance, the HAR–RQ–D and HAR–RQ–W models yield *MSE (QLIKE)* loss ratios of 0.864 (0.917) and 0.861 (0.902), respectively. While the HAR–RQ–W performs better than the more general HAR–RQ specification in Equation (46), a finding consistent with the work of [Bollerslev et al. \(2016\)](#), it does not outperform the weekly implied variance.

From a theoretical standpoint, the noise in the realized variance series should be larger at lower sampling frequencies ([Barndorff-Nielsen, 2002](#)). Thus, by sampling the data at finer frequencies, one should be able to dampen the effect of the noise. This insight motivates us to increase the sampling frequency from 5-minute to 1-minute and repeat our out-of-sample analysis. Table 6 points to the same conclusion as Table 5: the weekly implied variance yields the highest improvement in forecast accuracy.

5.2 The Method of Interpolation?

We obtain our implied variance series of constant maturity by applying a linear interpolation across maturities. It is thus interesting to assess the sensitivity of our results to the method of interpolation. To this end, we consider the nearest neighborhood interpolation method as an alternative approach. Essentially, we use the variance implied by option contracts of maturity closest to 1 week (month) in order to estimate the implied variance of the weekly (monthly) maturity. With this new time series, we repeat our out-of-sample analysis. Table 7 confirms our main results. The weekly implied variance is a strong predictor of the weekly realized variance. It reduces the forecasting error of the benchmark HAR model by 33.7% and 18.9% when considering the *MSE* and *QLIKE* loss functions, respectively. Moreover, the weekly implied variance outperforms the monthly implied variance as well as the recently proposed variance forecasting models.

5.3 The Implied Variance Estimator?

Our interest in the [Bakshi et al. \(2003\)](#) estimator is motivated by its robustness to jumps ([Du and Kapadia, 2013](#)). However, most existing studies, e.g. [Jiang and Tian \(2005\)](#) and [Taylor et al. \(2010\)](#), use the [Britten-Jones and Neuberger \(2000\)](#) estimator, making our results difficult to directly compare to those of the extant literature. As such, we use the numerical scheme presented in Section 2.1.b to implement the [Britten-Jones](#)

and Neuberger (2000) estimator of implied variance:²³

$$IV_t^\tau = \frac{2e^{rf_t \frac{\tau}{360}}}{\frac{\tau}{360}} \left(\int_0^{S_t} \frac{P_t(\tau, K)}{K^2} dK + \int_{S_t}^\infty \frac{C_t(\tau, K)}{K^2} dK \right) \quad (50)$$

where all variables are as previously discussed.

We use the resulting time series to repeat our out-of-sample analysis. Table 8 reports that the loss ratios associated with the implied variance series are generally higher than those based on the Bakshi et al. (2003) estimator (see Table 5). This is not surprising since the Britten-Jones and Neuberger (2000) estimator is biased in the presence of jumps (Du and Kapadia, 2013), resulting in larger forecast errors. However, most important for our purpose, the weekly implied variance series outperforms its monthly counterpart and all the competing models of the HAR family. This is true for both the *MSE* and the *QLIKE* loss functions. Overall, these results are consistent with our main findings.

5.4 A Longer Sample Period?

Our main analysis focuses on the period following the introduction of weekly options. However, one could extend the sample period back to January 1996 since, each month, we have one observation of a standard option with 7 days left to maturity (i.e. a standard option eventually becomes a weekly option once a month). We can thus check whether the main results hold in the extended sample. Note that, by extending the sample period

²³Generally, the literature directly uses the squared value of the volatility index (VIX) instead of computing the monthly implied variance series using the Britten-Jones and Neuberger (2000) formula and the numerical method presented in Section 2.1.b. It is interesting to compare the performance of the implied variance estimates obtained from these two approaches. By doing so, one can shed light on the impact of the numerical method on the results. Untabulated results show that the time series of the square of the VIX index and our own Britten-Jones and Neuberger (2000) series of monthly maturity are highly correlated and very similar. Empirically, the squared VIX series yields a *QLIKE* loss ratio of 1.251 that is comparable to the 1.217 figure based on the monthly implied variance using the Britten-Jones and Neuberger (2000) estimator (see Table 8). We thus conclude that the numerical method does not have a major impact on the results.

to 1996, we are forced to specify the regression to the monthly frequency.

Accordingly, we sample the implied variance series one week before the expiration of the closest to expiration standard option. We use this new dataset to repeat our main analyses. Tables A.1–A.4 of the online appendix confirm our main findings. In particular, the weekly implied variance outperforms the monthly implied variance and the recently proposed time series models of the HAR family. This finding holds both in- and out-of-sample.

5.5 A Longer Forecasting Horizon?

Up to this point, our results show that, if one wants to predict the realized variance over the next week, it is better to use options with the same maturity as the forecasting horizon as opposed to simply using monthly options, the usual practice in the literature. Naturally, one may wonder: is this a more general phenomenon? For instance, does the quarterly implied variance outperform the monthly implied variance when it comes to predicting the quarterly realized variance? Although this is not the main goal of this study, it is interesting to explore this possibility.²⁴

We compute the (annualized) quarterly realized variance as follows:

$$RV_{t+90}^q = 4 \times \sum_{j=0}^{N_{t+90}^q-1} \sum_{k=1}^m r_{t+90-j,k}^2 \quad (51)$$

where RV_{t+90}^q is the (annualized) quarterly realized variance for the quarter ending on day $t + 90$. The number 4 indicates that the realized variance estimate is annualized. N_{t+90}^q indicates the number of trading days during the quarter ending on day $t + 90$.

²⁴We thank the reviewer for suggesting this analysis.

Next, we estimate the following variance forecasting regression:

$$RV_{t+90}^q = \alpha + \beta IV_t^x + \epsilon_{t+90} \quad (52)$$

where α is the intercept. β denotes the slope parameter. IV_t^x is the implied variance on day t of time to maturity x , where x can be the quarterly (q) or monthly (m) maturity. ϵ_{t+90} is the residual of the regression at $t + 90$.

Given the large amount of overlap between consecutive daily observations of the realized variance, we sample all data at the quarterly frequency to obtain non-overlapping data samples. By taking this step, we address concerns related to the overlapping observation biases that typically plague long-horizon regressions. Unfortunately, this choice means that we have a very limited sample of independent observations. We attempt to mitigate this issue by extending the sample period to 1996 as in Section 5.4. Nonetheless, we should point out that the results of the in-sample analysis should be interpreted cautiously.²⁵ Table A.5 of the online appendix shows that the quarterly implied variance yields an *Adj R*² of 17.8%, which is slightly higher than that of the monthly implied variance.

Our working hypothesis is that the quarterly implied variance contains all relevant information to predict the quarterly realized variance. Consequently, the orthogonal component of the monthly implied variance with respect to the quarterly implied variance should not contain information about the future quarterly realized variance. This insight motivates the following encompassing model:

$$RV_{t+90}^q = \alpha + \beta IV_t^q + \gamma \hat{\nu}_t^{m/q} + \epsilon_{t+90} \quad (53)$$

²⁵Another implication of the limited sample size is that we are not able to carry out a reliable out-of-sample analysis.

where α is the intercept. β and γ are the slope parameters. IV_t^q is the implied variance on day t of quarterly time to maturity. $\hat{\nu}_t^{m/q}$ is the estimated residual at time t of the regression of the monthly implied variance on a constant and the quarterly implied variance (see Equation (54)):

$$IV_t^m = \phi_0 + \phi_1 IV_t^q + \nu_t^{m/q} \quad (54)$$

The results presented in the last row of Table A.5 of the online appendix are consistent with our main hypothesis. Namely, the orthogonalized monthly implied variance does not add to the information content of the quarterly implied variance.

6 Conclusion

We exploit the launch of weekly options on the S&P 500 index to analyze the information content of short-term options for short-term realized variance. Our results reveal that the weekly implied variance is a powerful predictor of future weekly realized variance. Out-of-sample, the weekly implied variance outperforms not only the monthly implied variance but also the HAR model and its recent extensions.

Our evidence carries implications for both academics and practitioners. For practitioners, it would be interesting for the Chicago Board of Options Exchange (CBOE) to compute and disseminate the time series of the weekly implied volatility index. This series, which would sit alongside the popular 30-day volatility index, would be useful for market participants to better gauge and manage short-term risk. For academics, our results suggest that one would benefit from not discarding short-term options on the grounds that they are illiquid and thus uninformative. Our analysis clearly shows that

these options are more informative about future short-term variance than the monthly options that have been analyzed hitherto.

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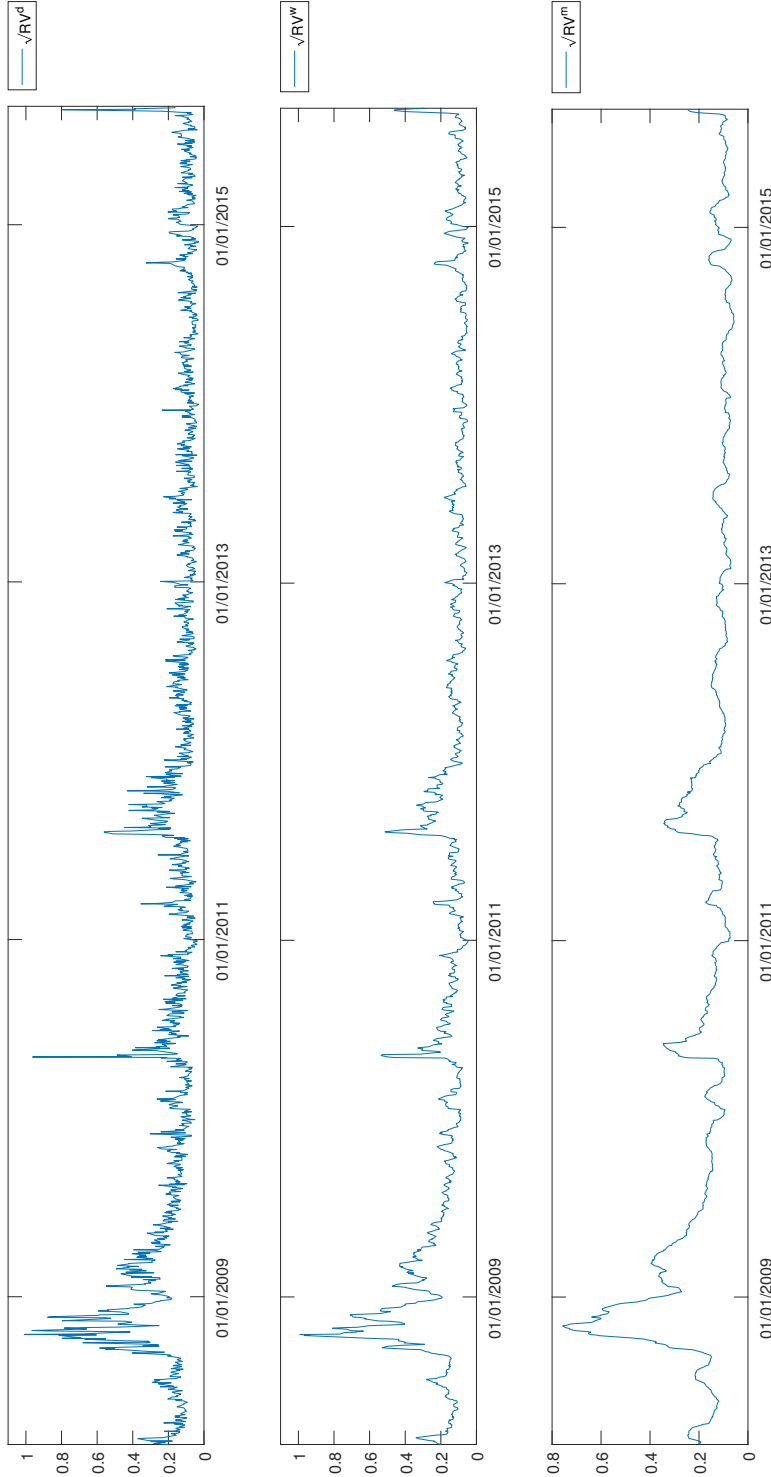


Figure 1: The Dynamics of \sqrt{RV}

This figure plots the time series dynamics of the square root of the realized variance of daily (top panel), weekly (middle panel) and monthly (bottom panel) horizons, respectively. The horizontal axis shows the observation date, while the vertical axis indicates the (annualized) values of the series.

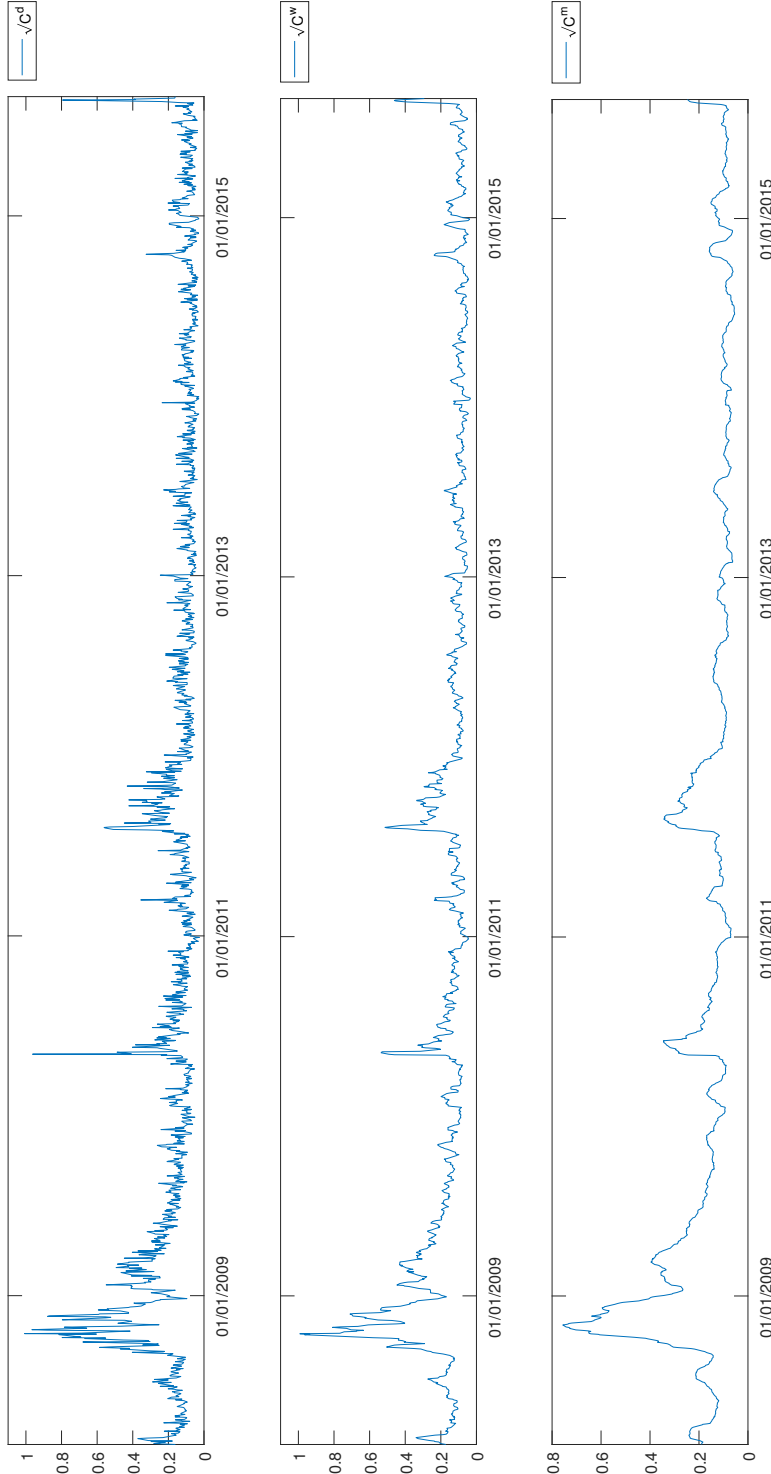


Figure 2: The Dynamics of \sqrt{C}

This figure plots the time series dynamics of the square root of the significant continuous variation of daily (top panel), weekly (middle panel) and monthly (bottom panel) horizons, respectively. The horizontal axis shows the observation date, while the vertical axis indicates the (annualized) values of the series.

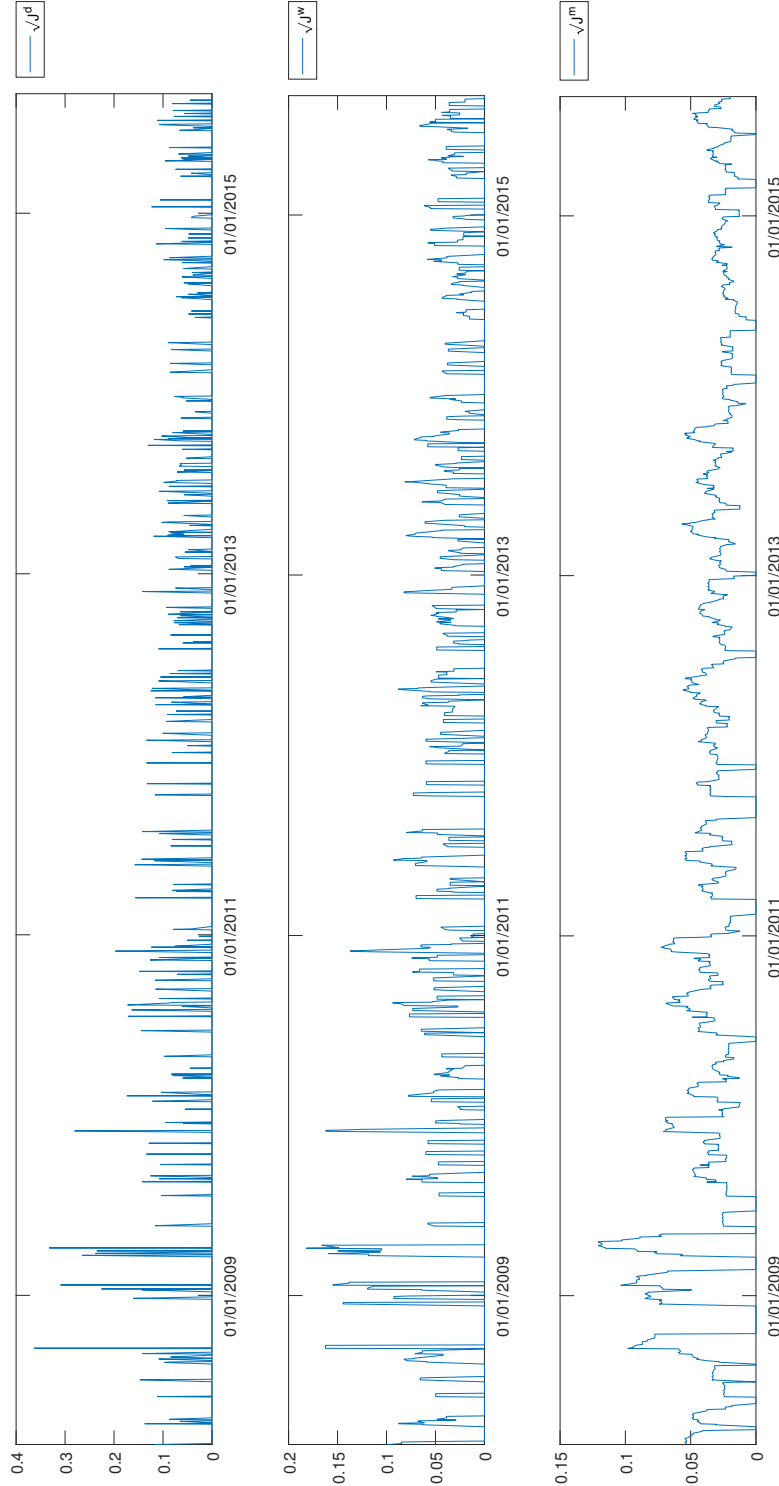


Figure 3: The Dynamics of \sqrt{J}

This figure plots the time series dynamics of the square root of the significant jump variation of daily (top panel), weekly (middle panel) and monthly (bottom panel) horizons, respectively. The horizontal axis shows the observation date, while the vertical axis indicates the (annualized) values of the series.

Table 1: Descriptive Statistics

This table presents key summary statistics. IV^w and IV^m denote the (annualized) implied variance of weekly and monthly horizons, respectively. RV^d , RV^w and RV^m denote the (annualized) realized variance of daily, weekly and monthly horizons, respectively. VRP^w and VRP^m are the weekly and monthly variance risk premia, respectively. Similar to [Bollerslev et al. \(2009\)](#), the variance risk premium of a given maturity is defined as the difference between the implied variance of that maturity and the contemporaneously computed realized variance of the same maturity. Returns data are sampled at the 5-minute frequency. *Mean* is the average value of the daily time series of the variable [name in row]. *Std*, *Skew* and *Kurt* denote the standard deviation, skewness and kurtosis of the variable [name in row], respectively.

	<i>Mean</i>	<i>Std</i>	<i>Skew</i>	<i>Kurt</i>
$\sqrt{IV^w}$	21.345%	11.850%	2.404	10.218
$\sqrt{IV^m}$	21.837%	10.996%	2.257	9.088
$\sqrt{RV^d}$	15.177%	11.429%	2.967	15.388
$\sqrt{RV^w}$	15.633%	11.121%	2.903	14.288
$\sqrt{RV^m}$	15.957%	10.635%	2.740	12.146
VRP^w	2.280%	4.659%	0.506	40.845
VRP^m	2.300%	3.107%	1.109	29.283

Table 2: In-Sample Results: Implied Variance

This table summarizes the results of regressions of the daily time series of the (annualized) weekly realized variance on a constant and the lagged forecasting variable(s) [name in column]. α denotes the intercept parameter. IV^w and IV^m are the [Bakshi et al. \(2003\)](#) weekly and monthly implied variances, respectively. The last two rows show the results from a multiple regression of the weekly realized variance on a constant, the lagged weekly implied variance and the lagged orthogonal component of the monthly implied variance (i.e. the estimated residual series from a regression of the monthly implied variance on a constant and the weekly implied variance). We present in parentheses the [Newey and West \(1987\)](#) t -statistics with 10 lags. $Adj R^2$ is the adjusted R -squared of the regression model. Returns are sampled at the 5-minute frequency.

α	IV^w	IV^m	$Adj R^2$
-0.005 (-1.376)	0.713 (7.240)		0.649
-0.010 (-2.140)		0.785 (6.676)	0.616
-0.005 (-1.338)	0.713 (7.025)	0.137 (0.631)	0.650

Table 3: Continuous vs. Jump Variation

This table dissects the source of the predictability of the weekly realized variance. Panel A presents the results of regressions of the daily time series of the (annualized) significant weekly continuous variation on a constant and the lagged variable(s) [name in column]. Panel B summarizes the results of the regression of the daily time series of the (annualized) significant weekly jump variation on a constant and the forecasting variable(s) [name in column]. α denotes the intercept parameter. IV^w and IV^m are the Bakshi et al. (2003) weekly and monthly implied variances, respectively. The last two rows of Panel A (Panel B) present the results from a multiple regression of the significant weekly continuous (jump) variation on a constant, the lagged weekly implied variance and the lagged orthogonal component of the monthly implied variance. The latter corresponds to the residual series from a regression of the monthly implied variance on a constant and the weekly implied variance. We present in parentheses the Newey–West t -statistics with 10 lags. $Adj R^2$ is the adjusted R -squared of the regression. Returns are sampled at the 5-minute frequency.

Panel A: Significant Continuous Variation

α	IV^w	IV^m	$Adj R^2$
-0.007 (-1.728)	0.695 (6.924)		0.636
-0.011 (-2.424)		0.764 (6.455)	0.602
-0.007 (-1.689)	0.695 (6.755)	0.119 (0.535)	0.637

Panel B: Significant Jump Variation

α	IV^w	IV^m	$Adj R^2$
0.002 (6.280)	0.018 (3.121)		0.103
0.001 (4.264)		0.021 (2.840)	0.109
0.002 (6.151)	0.018 (3.031)	0.018 (1.026)	0.108

Table 4: In-Sample Results: Time Series Models

This table summarizes the results of regressions of the daily time series of the (annualized) weekly realized variance on a constant and the lagged forecasting variable(s) [name in row]. α denotes the intercept parameter. RV^d , RV^w and RV^m denote the (annualized) daily, weekly and monthly realized variance series, respectively. C and J indicate the (annualized) significant continuous and (annualized) significant jump variations, respectively. The associated superscripts indicate that we compute these quantities for the daily (d), weekly (w) and monthly (m) horizons, respectively. SV^{d+} and SV^{d-} are the (annualized) positive and negative daily semivariances, respectively. SV^{w+} and SV^{w-} denote the (annualized) positive and negative weekly semivariances, respectively. SV^{m+} and SV^{m-} are the (annualized) positive and negative monthly semivariances, respectively. We present in parentheses the [Newey and West \(1987\)](#) t -statistics with 10 lags. $Adj R^2$ is the adjusted R -squared of the regression. Returns are sampled at the 5-minute frequency.

	HAR	$CHAR$	$HAR - J$	$HAR - C - J$	$SHAR$
α	0.005 (2.592)	0.007 (3.271)	0.005 (2.816)	0.002 (0.464)	0.006 (2.620)
RV^d	0.342 (2.650)		0.343 (2.607)		
RV^w	0.216 (3.254)		0.215 (3.214)		
RV^m	0.311 (3.679)		0.310 (3.669)		
C^d		0.340 (2.603)		0.338 (2.705)	
C^w		0.229 (3.387)		0.219 (2.938)	
C^m		0.299 (3.514)		0.298 (3.483)	
J^d			-0.084 (-0.397)	0.443 (3.070)	
J^w				-1.911 (-1.255)	
J^m				5.292 (1.297)	
SV^{d+}					0.075 (0.704)
SV^{d-}					0.482 (2.851)
SV^{w+}					-0.081 (-0.142)
SV^{w-}					0.577 (1.085)
SV^{m+}					0.084 (0.076)
SV^{m-}					0.546 (0.543)
$Adj R^2$	0.598	0.596	0.598	0.608	0.607

Table 5: Out-of-Sample Results

This table presents the ratio of the average loss based on the function [name in row] of the model [name in column] over that of the *HAR* model. We use a rolling window of 4 years of daily data to estimate the parameters of the forecasting models. *HAR* denotes the forecasting model that uses the daily, weekly and monthly lagged realized variance series to predict the weekly realized variance. *IVW* and *IVM* are the forecasting models based on the [Bakshi et al. \(2003\)](#) weekly and monthly implied variances, respectively. *CHAR* is the continuous heterogeneous autoregressive model. *HAR – J* extends the *HAR* model by including the significant lagged daily jump variation. The *HAR – C – J* model decomposes the realized variances in the *HAR* model into their significant continuous and jump components. The *SHAR* model extends the *HAR* by splitting the lagged historical variance terms into the corresponding positive and negative semivariance components. *HAR – RQ* takes into account the heteroskedasticity of the measurement error in all three maturities of lagged realized variance. Similar to [Bollerslev et al. \(2016\)](#), we proxy the heteroskedasticity of the measurement error in the realized variance with the square root of the realized quarticity of corresponding maturity. Returns are sampled at the 5-minute frequency.

	<i>IVW</i>	<i>IVM</i>	<i>CHAR</i>	<i>HAR – J</i>	<i>HAR – C – J</i>	<i>SHAR</i>	<i>HAR – RQ</i>
<i>MSE</i>	0.797	0.955	1.012	1.001	0.998	0.887	0.836
<i>QLIKE</i>	0.870	1.049	1.022	1.000	1.016	0.951	0.939

Table 6: Out-of-Sample Results (1-Minute Sampling Frequency)

This table presents the ratio of the average loss based on the function [name in row] of the model [name in column] over that of the *HAR* model. We use a rolling window of 4 years of daily data to estimate the parameters of the forecasting models. *HAR* denotes the forecasting model that uses the daily, weekly and monthly lagged realized variance series to predict the future weekly realized variance. *IVW* and *IVM* are the forecasting models based on the [Bakshi et al. \(2003\)](#) weekly and monthly implied variances, respectively. *CHAR* is the continuous heterogeneous autoregressive model. *HAR – J* extends the *HAR* model by including the significant lagged daily jump variation. The *HAR – C – J* model decomposes the realized variances in the *HAR* model into their significant continuous and jump components. The *SHAR* model extends the *HAR* by splitting the lagged historical variance terms into the corresponding positive and negative semivariance components. *HAR – RQ* takes into account the heteroskedasticity of the measurement error in all three maturities of lagged realized variance. Similar to [Bollerslev et al. \(2016\)](#), we proxy the heteroskedasticity of the measurement error in the realized variance with the square root of the realized quarticity of corresponding maturity. Returns are sampled at the 1-minute frequency.

	<i>IVW</i>	<i>IVM</i>	<i>CHAR</i>	<i>HAR – J</i>	<i>HAR – C – J</i>	<i>SHAR</i>	<i>HAR – RQ</i>
<i>MSE</i>	0.791	1.013	0.936	1.014	1.002	1.021	0.883
<i>QLIKE</i>	0.897	1.145	1.159	1.024	0.970	0.948	0.962

Table 7: Out-of-Sample Results (Nearest)

This table presents the ratio of the average loss based on the function [name in row] of the model [name in column] over that of the *HAR* model. We use a rolling window of 4 years of daily data to estimate the parameters of the forecasting models. *HAR* denotes the forecasting model that uses the daily, weekly and monthly lagged realized variance series to predict the future weekly realized variance. *IVW* and *IVM* are the forecasting models based on the [Bakshi et al. \(2003\)](#) weekly and monthly implied variances computed using options of maturities nearest to the weekly and monthly horizons, respectively. *CHAR* is the continuous heterogeneous autoregressive model. *HAR – J* extends the *HAR* model by including the significant lagged daily jump variation. The *HAR – C – J* model decomposes the realized variances in the *HAR* into their significant continuous and jump components. The *SHAR* model extends the *HAR* by splitting the lagged historical variance terms into the corresponding positive and negative semivariance components. *HAR – RQ* takes into account the heteroskedasticity of the measurement error in all three maturities of lagged realized variance. Similar to [Bollerslev et al. \(2016\)](#), we proxy the heteroskedasticity of the measurement error in the realized variance with the square root of the realized quarticity of corresponding maturity. Returns are sampled at the 5-minute frequency.

	<i>IVW</i>	<i>IVM</i>	<i>CHAR</i>	<i>HAR – J</i>	<i>HAR – C – J</i>	<i>SHAR</i>	<i>HAR – RQ</i>
<i>MSE</i>	0.763	0.861	1.012	1.001	0.998	0.887	0.836
<i>QLIKE</i>	0.811	0.915	1.022	1.000	1.016	0.951	0.939

Table 8: Out-of-Sample Results (Britten-Jones and Neuberger, 2000)

This table presents the ratio of the loss function [name in row] of the model [name in column] over that of the *HAR* model. We use a rolling window of 4 years of daily data to estimate the parameters of the forecasting models. *HAR* denotes the forecasting model that uses the daily, weekly and monthly lagged realized variance series to predict the future weekly realized variance. We compute the realized variance estimates with 1-minute return data. *IVW* and *IVM* are the forecasting models based on the Britten-Jones and Neuberger (2000) weekly and monthly implied variances, respectively. *CHAR* is the continuous heterogeneous autoregressive model. *HAR – J* extends the *HAR* model by including the significant lagged daily jump variation. The *HAR – C – J* model decomposes the realized variances in the *HAR* into their significant continuous and jump components. The *SHAR* model extends the *HAR* by splitting the lagged historical variance terms into the corresponding positive and negative semivariance components. *HAR – RQ* takes into account the heteroskedasticity of the measurement error in all three maturities of lagged realized variance. Similar to Bollerslev et al. (2016), we proxy the heteroskedasticity of the measurement error in the realized variance with the square root of the realized quarticity of corresponding maturity. Returns are sampled at the 5-minute frequency.

	<i>IVW</i>	<i>IVM</i>	<i>CHAR</i>	<i>HAR – J</i>	<i>HAR – C – J</i>	<i>SHAR</i>	<i>HAR – RQ</i>
<i>MSE</i>	0.805	1.121	1.020	1.010	0.999	0.887	0.836
<i>QLIKE</i>	0.891	1.217	1.004	0.993	1.015	0.951	0.939

Appendix to

**“The Information Content of
Short-Term Options”**

Not Intended for Publication

Will be Provided as Online Appendix

Table A.1: Descriptive Statistics (Extended Sample)

This table presents key summary statistics based on the extended sample covering the period starting from January 1996 to August 2015. IV^w and IV^m denote the (annualized) implied variance of weekly and monthly horizons, respectively. RV^d , RV^w and RV^m denote the (annualized) realized variance of daily, weekly and monthly horizons, respectively. VRP^w and VRP^m are the weekly and monthly variance risk premia, respectively. Similar to [Bollerslev et al. \(2009\)](#), the variance risk premium of a given maturity is defined as the difference between the implied variance of that maturity and the contemporaneously computed realized variance of the same maturity. Returns data are sampled at the 5-minute frequency. *Mean* is the average value of the daily time series of the variable [name in row]. *Std*, *Skew* and *Kurt* denote the standard deviation, skewness and kurtosis of the variable [name in row], respectively.

	<i>Mean</i>	<i>Std</i>	<i>Skew</i>	<i>Kurt</i>
$\sqrt{IV^w}$	19.871%	9.950%	2.616	14.336
$\sqrt{IV^m}$	20.951%	9.434%	2.606	14.322
$\sqrt{RV^d}$	13.728%	10.663%	4.145	27.600
$\sqrt{RV^w}$	14.776%	9.033%	2.941	15.365
$\sqrt{RV^m}$	14.982%	8.287%	3.001	16.716
VRP^w	1.938%	3.301%	2.664	23.254
VRP^m	2.348%	2.627%	2.661	13.881

Table A.2: In-Sample Results: Implied Variance (Extended Sample)

This table summarizes the results of regressions of the monthly time series of the (annualized) weekly realized variance on a constant and the lagged forecasting variable(s) [name in column]. α denotes the intercept parameter. IV^w and IV^m denote the (annualized) implied variance of weekly and monthly horizons, respectively. The last two rows show the results from a multiple regression of the weekly realized variance on a constant, the lagged weekly implied variance and the lagged orthogonalized monthly implied variance (i.e. the estimated residual series from a regression of the monthly implied variance on a constant and the weekly implied variance). We present in parentheses the [Newey and West \(1987\)](#) *t*-statistics with 10 lags. *Adj R²* is the adjusted *R*-squared of the regression model. Returns are sampled at the 5-minute frequency. The extended sample covers the period from January 1996 to August 2015.

α	IV^w	IV^m	<i>Adj R²</i>
-0.001 (-0.396)	0.563 (21.739)		0.815
-0.003 (-2.099)		0.576 (20.731)	0.769
-0.001 (-0.391)	0.563 (21.340)	-0.047 (-0.226)	0.814

Table A.3: In-Sample Results: Time Series Models (Extended Sample)

This table summarizes the results of regressions of the monthly time series of the (annualized) weekly realized variance on a constant and the lagged forecasting variable(s) [name in row]. α denotes the intercept parameter. RV^d , RV^w and RV^m denote the (annualized) daily, weekly and monthly realized variance series, respectively. C and J indicate the (annualized) significant continuous and (annualized) significant jump variations, respectively. The associated superscripts indicate that we compute these quantities for the daily (d), weekly (w) and monthly (m) horizons, respectively. SV^{d+} and SV^{d-} are the (annualized) positive and negative daily semivariances, respectively. SV^{w+} and SV^{w-} denote the (annualized) positive and negative weekly semivariances, respectively. SV^{m+} and SV^{m-} are the (annualized) positive and negative monthly semivariances, respectively. We present in parentheses the [Newey and West \(1987\)](#) t -statistics with 10 lags. $Adj R^2$ is the adjusted R -squared of the regression. Returns are sampled at the 5-minute frequency. The extended sample covers the period from January 1996 to August 2015.

	HAR	$CHAR$	$HAR - J$	$HAR - C - J$	$SHAR$
α	0.010 (3.914)	0.010 (4.000)	0.010 (3.869)	0.009 (3.503)	0.009 (4.221)
RV^d	0.291 (3.938)		0.291 (3.935)		
RV^w	0.013 (0.101)		0.012 (0.098)		
RV^m	0.288 (2.516)		0.289 (2.529)		
C^d		0.251 (2.785)		0.278 (3.131)	
C^w		0.124 (0.623)		0.110 (0.568)	
C^m		0.236 (1.915)		0.198 (1.615)	
J^d			-0.283 (-0.620)	0.316 (0.689)	
J^w				-0.620 (-2.130)	
J^m				1.701 (1.905)	
SV^{d+}					-0.050 (-0.470)
SV^{d-}					0.408 (4.049)
SV^{w+}					0.072 (0.263)
SV^{w-}					0.358 (1.187)
SV^{m+}					0.287 (0.427)
SV^{m-}					0.156 (0.211)
$Adj R^2$	0.757	0.757	0.756	0.764	0.778

Table A.4: Out-of-Sample Results (Extended Sample)

This table presents the ratio of the average loss based on the function [name in row] of the model [name in column] over that of the *HAR* model. We use a rolling window of 4 years of monthly data to estimate the parameters of the forecasting models. *HAR* denotes the forecasting model that uses the daily, weekly and monthly lagged realized variance series to predict the future weekly realized variance. *IVW* and *IVM* are the forecasting models based on the [Bakshi et al. \(2003\)](#) weekly and monthly implied variances, respectively. *CHAR* is the continuous heterogeneous autoregressive model. *HAR – J* extends the *HAR* model by including the significant lagged daily jump variation. The *HAR – C – J* model decomposes the realized variances in the *HAR* into their significant continuous and jump components. The *SHAR* model extends the *HAR* by splitting the lagged historical variance terms into the corresponding positive and negative semivariance components. *HAR – RQ* takes into account the heteroskedasticity of the measurement error in all three maturities of lagged realized variance. Similar to [Bollerslev et al. \(2016\)](#), we proxy the heteroskedasticity of the measurement error in the realized variance with the square root of the realized quarticity of corresponding maturity. Returns are sampled at the 5-minute frequency. The extended sample covers the period from January 1996 to August 2015.

	<i>IVW</i>	<i>IVM</i>	<i>CHAR</i>	<i>HAR – J</i>	<i>HAR – C – J</i>	<i>SHAR</i>	<i>HAR – RQ</i>
<i>MSE</i>	0.885	0.941	1.012	1.013	1.085	1.059	1.158
<i>QLIKE</i>	0.841	0.940	1.023	1.162	1.531	1.042	1.226

Table A.5: In-Sample Results: Implied Variance (Extended Sample and Quarterly Horizon)

This table summarizes the results of regressions of the non-overlapping time series of the (annualized) quarterly realized variance on a constant and the lagged forecasting variable(s) [name in column]. α denotes the intercept parameter. IV^q and IV^m are the [Bakshi et al. \(2003\)](#) quarterly and monthly implied variances, respectively. We present in parentheses the [Newey and West \(1987\)](#) t -statistics with 3 lags. $Adj R^2$ is the adjusted R -squared of the regression model. Returns are sampled at the 5-minute frequency. The first two sets of regressions are univariate. The third regression consists in regressing the quarterly realized variance on a constant, the lagged quarterly implied variance and the lagged orthogonal component of the monthly implied variance with respect to the lagged quarterly implied variance. To be more specific, the orthogonal monthly implied variance is the estimated residual from the regression of the monthly implied variance on a constant and the contemporaneous quarterly implied variance. The extended sample covers the period from January 1996 to August 2015.

α	IV^q	IV^m	$Adj R^2$
0.013 (2.553)	0.289 (12.327)		0.178
0.016 (2.944)		0.245 (9.818)	0.172
0.013 (2.572)	0.289 (12.504)	-0.026 (-0.070)	0.167